Problem Example

A bead lies on a frictionless hoop of radius $R$ that rotates around a vertical diameter with constant angular frequency $\omega$, as shown below.

What should $\omega$ be so that the bead maintains the same position on the hoop, at an angle $\beta$ with respect to the vertical?

Solution: The hoop is rotating at a constant angular frequency $\omega$. The bead is traveling in a circle perpendicular to the hoop with radius $R \sin \beta$ as shown in figures.
We know for general motion in polar coordinates

\[ \mathbf{a} = \left( \ddot{r} - r\dot{\theta}^2 \right) \mathbf{\hat{r}} - \left( r\ddot{\theta} + 2\dot{r}\dot{\theta} \right) \mathbf{\hat{\theta}} \]  

(1)

We also know \( \dot{\theta} = \omega \) and \( \ddot{\theta} = \dot{\omega} = 0 \) since \( \omega \) is constant.